International Journal of Multiphase Flow 34 (2008) 865–868

Contents lists available at ScienceDirect

International Journal of Multiphase Flow

journal homepage: www.elsevier.com/locate/ijmulflow

Acceleration of heavy particles in isotropic turbulence

Leonid I. Zaichik [a,*], Vladimir M. Alipchenkov [b]

[a] Nuclear Safety Institute of the Russian Academy of Sciences, B. Tulskaya 52, 115191 Moscow, Russia
[b] Institute for High Temperatures of the Russian Academy of Sciences, Krasnokazarmennaya 17a, 111116 Moscow, Russia

ARTICLE INFO

Article history:
Received 27 July 2007
Received in revised form 24 December 2007
Available online 17 March 2008

Keywords:
Acceleration statistics
Particle inertia
Heavy particles
Isotropic turbulence

ABSTRACT

The paper concerns the effect of particle inertia on acceleration statistics. A simple analytical model for predicting the acceleration of heavy particles suspended in an isotropic homogeneous turbulent flow field is developed. This model is capable of describing the influence of both Stokes and Reynolds numbers on the particle acceleration variance. Comparisons of model predictions with numerical simulations are presented.

1. Introduction

Transport and dispersion of small heavy particles in turbulent fluid occur in many natural and industrial processes. Due to the practical and theoretical importance of the problem, numerous works have been devoted to investigating the velocity and displacement statistics of heavy inertial particles dispersed in isotropic homogeneous turbulence (e.g., see Reeks, 1977; Pismen and Nir, 1978; Squires and Eaton, 1991; Elghobashi and Truesdell, 1992; Wang and Stock, 1993; Pozorski and Minier, 1998; Mashayek, 1999; Derevich, 2001). At the same time, a much lesser attention has been paid to the acceleration statistics of heavy particles in turbulent flow. Recently, this gap in knowledge has been filled by Bec et al. (2006). In this paper, a detailed direct numerical simulation of the behaviour of particle acceleration at varying both Stokes and Reynolds numbers was performed.

The present paper is focused on deriving an analytical model for predicting the effect of particle inertia on the acceleration variance and autocorrelation function. Hereafter the particles are considered as heavy if their material density is much large than that of the fluid. Moreover, the particle size is assumed to be much less as compared to the Kolmogorov length microscale.

2. Velocity correlations

Before proceeding to acceleration statistics, we will define velocity correlations. In an isotropic homogeneous stationary turbulent flow field, the Lagrangian fluctuating velocity correlations are defined as

$$B_{Lij}(\tau) = \langle u'_i(\mathbf{x},t)u'_j(\mathbf{R}(t-\tau),t-\tau)|\mathbf{R}(t)=\mathbf{x}\rangle = u'^2\Psi_L(\tau)\delta_{ij}, \tag{1}$$

where $\mathbf{R}$ is the position vector of a fluid element along its path, $u'^2 \equiv \langle u'_k u'_k\rangle/3$ is the intensity of fluid velocity fluctuations, and $\Psi_L(\tau)$ is the Lagrangian autocorrelation function. Hence and henceforth the angle brackets symbolize averaging over the ensemble of samples of a random fluid velocity field. The most widespread dependence for approximating $\Psi_L(\tau)$ is the exponential function

$$\Psi_L(\tau) = \exp(-\tau T_L^{-1}) \tag{2}$$

with $T_L \equiv \int_0^\infty \Psi_L(\tau)d\tau$ being the Lagrangian integral timescale. As is clear, (2) contains the sole timescale T_L which is a measure of the large-scale energy-containing eddies. Nonetheless, the function (2) describes experimental data and DNS results reasonably well (except for the region of small values of τ) when using an appropriate dependence of T_L on Reynolds number. In the vicinity of $\tau = 0$, the behaviour of (2) is incorrect since $\Psi'_L(0) \neq 0$.

The Lagrangian fluid velocity correlations measured along heavy particle trajectories (seen by particles) are defined as

$$B_{Lpij}(\tau) = \langle u'_i(\mathbf{x},t)u'_j(\mathbf{R}_p(t-\tau),t-\tau)|\mathbf{R}_p(t)=\mathbf{x}\rangle = u'^2\Psi_{Lpij}(\tau), \tag{3}$$

where $\mathbf{R}_p$ is the particle position vector, and $\Psi_{Lpij}(\tau)$ is the fluid velocity autocorrelation tensorial function seen by particles. It is worth emphasizing that, due to the so-called crossing-trajectories effect induced by the mean velocity drift between the particulate and fluid phases (Csanady, 1963), the fluid velocity correlations seen by particles may be anisotropic even in isotropic turbulence.

* Corresponding author. Tel.: +7 495 955 2394; fax: +7 495 958 1151.
E-mail address: zaichik@ibrae.ac.ru (L.I. Zaichik).

0301-9322/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijmultiphaseflow.2008.02.008

Thus, the Lagrangian fluid velocity correlations seen by particles (3) are characterized by the autocorrelation tensor $\Psi_{Lpij}(\tau)$ as contrasted to the Lagrangian correlations measured along fluid element trajectories (1), which are quantified by the scalar autocorrelation function $\Psi_L(\tau)$.

When using matrix notation, $\Psi_{Lpij}(\tau)$ can be also approximated by the exponential function

$$\Psi_{Lp}(\tau) = \exp(-\tau \mathbf{T}_{Lp}^{-1}) \tag{4}$$

with $T_{Lpij} \equiv \int_0^\infty \Psi_{Lpij}(\tau)d\tau$ being the Lagrangian fluid velocity timescale tensor seen by particles. This timescale tensor should take into consideration the inertia, crossing-trajectories, and continuity effects (Reeks, 1977; Wang and Stock, 1993). In the absence of the crossing-trajectories effect, T_{Lpij} becomes isotropic, that is, $T_{Lpij} = T_{Lp}\,\delta_{ij}$. This effect may be neglected when the mean velocity drift, which is caused for example by gravity, is small as compared with the fluid fluctuating velocity (i.e., $\tau_p g \ll u'$, where τ_p is the particle response time, and g is the gravity acceleration). Even if the crossing-trajectories effect is negligible, the eddy-particle interaction timescale, T_{Lp}, coincides with the Lagrangian fluid turbulence timescale, T_L, only in the limit of zero-inertia particles, when the dynamic behaviour of particles is equivalent to the motion of fluid elements. For heavy particles, T_{Lp} may differ from T_L and, depending on particle inertia and turbulence structure parameters, T_{Lp}/T_L can be considerably greater than unity (Oesterlé and Zaichik, 2006).

The Lagrangian particle velocity correlations are written as

$$B_{pij}(\tau) = \langle v_i'(\mathbf{x},t)v_j'(\mathbf{R}_p(t-\tau),t-\tau)|\mathbf{R}_p(t)=\mathbf{x}\rangle = \langle v_i'v_k'\rangle \Psi_{pkj}(\tau), \tag{5}$$

where $\langle v_i'v_j'\rangle$ is the particulate stress tensor, and $\Psi_{pij}(\tau)$ is the particle fluctuating velocity autocorrelation tensor. In should be noted that, due to the crossing-trajectories effect, both $\langle v_i'v_j'\rangle$ and $\Psi_{pij}(\tau)$ may be anisotropic even in isotropic turbulence.

The motion of a heavy particle is governed by the equation

$$\frac{d\mathbf{v}_p}{dt} = \frac{\mathbf{u}(\mathbf{R}_p,t) - \mathbf{v}_p}{\tau_p} + \mathbf{F}, \tag{6}$$

where $\mathbf{u}(\mathbf{R}_p,t)$ is the velocity of the carrier fluid at a point $\mathbf{x} = \mathbf{R}_p(t)$, and $\mathbf{F}$ is a body force acting on a particle (e.g., gravity).

Eq. (6) produces the following equation for the particle velocity correlations (5):

$$\frac{d^2 B_{pij}}{d\tau^2} - \frac{B_{pij}}{\tau_p^2} = -\frac{B_{Lpij}}{\tau_p^2}. \tag{7}$$

By making use of matrix notation, the solution of (7) that satisfies the boundary conditions

$$\frac{dB_{pij}}{d\tau} = 0 \quad \text{for } \tau = 0, \quad B_{pij} \to 0 \quad \text{for } \tau \to \infty$$

is given by

$$\mathbf{B}_p(\tau) = \frac{u'^2}{2\tau_p}\int_0^\infty \left[\exp\left(-\frac{|\tau+\xi|}{\tau_p}\mathbf{I}\right) + \exp\left(-\frac{|\tau-\xi|}{\tau_p}\mathbf{I}\right)\right]\mathbf{\Psi}_{Lp}(\xi)d\xi \tag{8}$$

with $\mathbf{I}$ being the unit matrix. Expression (8) was first obtained by Reeks (1977) immediately by integrating the particle motion Eq. (6). In accordance with Eq. (8), the kinetic particulate stresses are determined as

$$\langle v_i'v_j'\rangle = B_{pij}(0) = u'^2 f_{uij}. \tag{9}$$

The quantity

$$\mathbf{f}_u \equiv \frac{1}{\tau_p}\int_0^\infty \mathbf{\Psi}_{Lp}(\tau)\exp\left(-\frac{\tau}{\tau_p}\mathbf{I}\right)d\tau \tag{10}$$

measures a response of particles to velocity fluctuations of the carrier turbulent fluid, i.e., a coupling between the particulate and fluid phases.

If the Lagrangian autocorrelation function seen by particles is described by the exponential approximation (4), the particle velocity correlation tensor (8) takes the form

$$\begin{aligned}\mathbf{B}_p(\tau) = \frac{u'^2}{2}\Big\{&(\mathbf{I}+\tau_p\mathbf{T}_{Lp}^{-1})^{-1}\left[\exp(-\tau\mathbf{T}_{Lp}^{-1}) + \exp\left(-\frac{\tau}{\tau_p}\mathbf{I}\right)\right] \\ &+(\mathbf{I}-\tau_p\mathbf{T}_{Lp}^{-1})^{-1}\left[\exp(-\tau\mathbf{T}_{Lp}^{-1}) - \exp\left(-\frac{\tau}{\tau_p}\mathbf{I}\right)\right]\Big\},\end{aligned} \tag{11}$$

and the particle response tensor (10) becomes equal to

$$\mathbf{f}_u = (\mathbf{I}+\tau_p\mathbf{T}_{Lp}^{-1})^{-1}. \tag{12}$$

In the absence of the mean velocity drift between the particulate and fluid phases, the velocity correlation and response tensors become isotropic

$$\begin{aligned}B_{pij}(\tau) = B_p(\tau)\delta_{ij}, B_p(\tau) = \frac{u'^2}{2\tau_p}\int_0^\infty &\left[\exp\left(-\frac{|\tau+\xi|}{\tau_p}\right)\right. \\ &\left.+\exp\left(-\frac{|\tau-\xi|}{\tau_p}\right)\right]\Psi_{Lp}(\xi)d\xi,\end{aligned} \tag{13}$$

$$f_{uij} = f_u\delta_{ij}, f_u = \frac{1}{\tau_p}\int_0^\infty \Psi_{Lp}(\tau)\exp\left(-\frac{\tau}{\tau_p}\right)d\tau. \tag{14}$$

It is evident that, in this case, the particle velocity autocorrelations and kinetic stresses would be also isotropic

$$\begin{aligned}\Psi_{pij}(\tau) = \Psi_p(\tau)\delta_{ij}, \Psi_p(\tau) = \frac{1}{2\tau_p f_u}\int_0^\infty &\left[\exp\left(-\frac{|\tau+\xi|}{\tau_p}\right)\right. \\ &\left.+\exp\left(-\frac{|\tau-\xi|}{\tau_p}\right)\right]\Psi_{Lp}(\xi)d\xi,\end{aligned} \tag{15}$$

$$\langle v_i'v_j'\rangle = v'^2\delta_{ij}, v'^2 = f_u u'^2, \tag{16}$$

where $v'^2 \equiv \langle v_k'v_k'\rangle/3$ is the intensity of the particle velocity fluctuations.

When using the exponential approximation of the fluid velocity autocorrelation function seen by particles

$$\Psi_{Lp}(\tau) = \exp(-\tau T_{Lp}^{-1}), \tag{17}$$

the response coefficient (14) and the autocorrelation function (15) simplify to

$$f_u = \frac{T_{Lp}}{\tau_p + T_{Lp}}, \tag{18}$$

$$\begin{aligned}\Psi_p(\tau) = &\frac{1}{2}\left[\exp\left(-\frac{\tau}{T_{Lp}}\right) + \exp\left(-\frac{\tau}{\tau_p}\right)\right] \\ &+\frac{(T_{Lp}+\tau_p)}{2(T_{Lp}-\tau_p)}\left[\exp\left(-\frac{\tau}{T_{Lp}}\right) - \exp\left(-\frac{\tau}{\tau_p}\right)\right].\end{aligned} \tag{19}$$

The relation (18) that quantifies the particle-to-fluid velocity variance ratio was first derived by Chen (Hinze, 1975) who assumed $T_{Lp} = T_L$. It is well known that (18) is able to properly describe v'^2/u'^2 if T_{Lp} as a function of both Reynolds and Stokes numbers is taken into account as well as the vicinity of small particle inertia τ_p is excluded from consideration. At small values of τ_p, the behaviour of f_u is governed by the Taylor differential timescale rather than the Lagrangian integral one (Zaichik et al., 2003). To refine the behaviour of f_u at small values of τ_p, the two-scale bi-exponential approximation of the autocorrelation function proposed by Sawford (1991) can be applied

$$\Psi_{Lp}(\tau) = \frac{1}{2\Re}\left[(1+\Re)\exp\left(-\frac{\tau}{T_+}\right) - (1-\Re)\exp\left(-\frac{\tau}{T_-}\right)\right], \tag{20}$$

$$\Re = (1-2z^2)^{1/2}, \quad z = \frac{\tau_T}{T_{Lp}}, \quad T_+ = \frac{(1+\Re)T_{Lp}}{2}, \quad T_- = \frac{(1-\Re)T_{Lp}}{2}.$$

In (20), the influence of particle inertia is included only in the Lagrangian integral timescale, T_{Lp}, whereas the Taylor differential timescale, τ_T, is given by the convectional relation that does not allow for the inertia effect

$$\tau_T = \left(-\frac{2}{\Psi''_{\mathrm{L}}(0)}\right)^{1/2} = \left(\frac{2Re_\lambda}{15^{1/2}a_0}\right)^{1/2}\tau_k. \tag{21}$$

Here $Re_\lambda \equiv (15u'^4/\varepsilon\nu)^{1/2}$ is the Reynolds number based on the Taylor length microscale, and $\tau_k \equiv (\nu/\varepsilon)^{1/2}$ is the Kolmogorov time microscale, where ε is the turbulence dissipation rate and ν is the kinematic viscosity of the fluid. The quantity a_0 in (21) denotes the normalized magnitude of fluid acceleration fluctuations, and, due to intermittency, it is dependent on the Reynolds number (e.g., see Voth et al., 2002; Hill, 2002; Yeung et al., 2006). This Reynolds number dependence can be approximated as (Zaichik et al., 2003)

$$a_0 = \frac{a_{01} + a_{0\infty}Re_\lambda}{a_{02} + Re_\lambda}, \quad a_{01} = 11,\ a_{02} = 205,\ a_{0\infty} = 7. \tag{22}$$

The autocorrelation function (20) produces the following formulas for the response coefficient (14) and the autocorrelation function (15):

$$f_u = \frac{(2\tau_{\mathrm{p}} + z^2T_{\mathrm{Lp}})T_{\mathrm{Lp}}}{2\tau_{\mathrm{p}}^2 + 2\tau_{\mathrm{p}}T_{\mathrm{Lp}} + z^2T_{\mathrm{Lp}}^2}, \tag{23}$$

$$\begin{aligned}\Psi_{\mathrm{p}}(\tau) = \frac{1}{4\Re f_u}\Bigg\{ & \frac{(1+\Re)T_+}{\tau_{\mathrm{p}}+T_+}\left[\exp\left(-\frac{\tau}{T_+}\right)+\exp\left(-\frac{\tau}{\tau_{\mathrm{p}}}\right)\right] \\ & +\frac{(1+\Re)T_+}{T_+-\tau_{\mathrm{p}}}\left[\exp\left(-\frac{\tau}{T_+}\right)-\exp\left(-\frac{\tau}{\tau_{\mathrm{p}}}\right)\right] \\ & -\frac{(1-\Re)T_-}{\tau_{\mathrm{p}}+T_-}\left[\exp\left(-\frac{\tau}{T_-}\right)+\exp\left(-\frac{\tau}{\tau_{\mathrm{p}}}\right)\right] \\ & -\frac{(1-\Re)T_-}{T_--\tau_{\mathrm{p}}}\left[\exp\left(-\frac{\tau}{T_-}\right)-\exp\left(-\frac{\tau}{\tau_{\mathrm{p}}}\right)\right]\Bigg\}.\end{aligned} \tag{24}$$

It is clear that, as Re_λ increases, $z \to 0$, and hence, Eqs. (23) and (24) reduce, respectively, to Eqs. (18) and (19).

In Fig. 1, we present the comparison of the particle velocity autocorrelation functions (19) and (24) with DNS by Simonin et al. (2002) for various Stokes numbers at $Re_\lambda = 53$. The Stokes number is defined as $St_{\mathrm{E}} = \tau_{\mathrm{p}}/T_{\mathrm{E}}$ with T_{E} being the Eulerian integral timescale. The timescales T_{Lp} and T_{E} are determined by means of the following approximations (Oesterlé and Zaichik, 2006):

$$T_{\mathrm{Lp}} = T_{\mathrm{L}} + (T_{\mathrm{E}} - T_{\mathrm{L}})\left[\frac{St_{\mathrm{E}}}{1+St_{\mathrm{E}}} - \frac{0.9mSt_{\mathrm{E}}^2}{(1+St_{\mathrm{E}})^2(2+St_{\mathrm{E}})}\right],$$
$$T_{\mathrm{E}} = \frac{3(1+m)^2}{3+2m}T_{\mathrm{L}}, \tag{25}$$

where $m \equiv T_{\mathrm{E}}u'/L$ is the turbulence structure parameter, and L is the length macroscale. The structure parameter is taken as 0.3 that, according to (25), gives $T_{\mathrm{L}}/T_{\mathrm{E}} = 0.71$ what is close to the value of 0.68 obtained in Simonin et al. (2002). The Taylor timescale appearing in (24) is determined by (21) and (22).

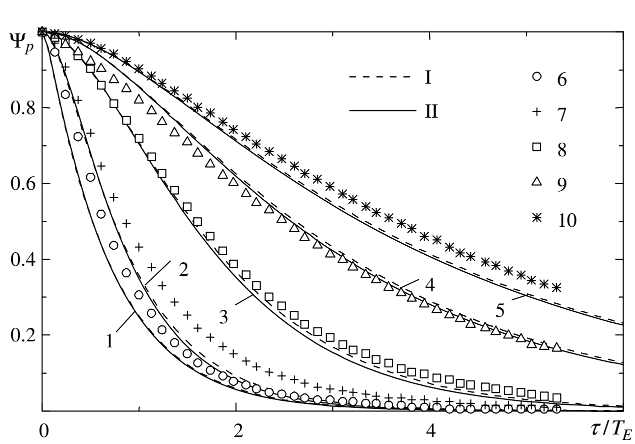

Fig. 1. The particle velocity autocorrelation function: I – (19); II – (24); 6–10 – Simonin et al. (2002); 1, 6 – $St_E = 0.04$; 2, 7 – $St_E = 0.2$; 3, 8 – $St_E = 1.0$; 4, 9 – $St_E = 2.3$; 5, 10 – $St_E = 3.3$.

As is seen from Fig. 1, the particle velocity autocorrelation function (19), which is relied on the exponential approximation (17), is in good agreement with the DNS results, and this is hardly distinguishable from the autocorrelation (24) based on the bi-exponential approximation (20). The most remarkable conclusion that can be drown from Fig. 1 consists in growing the Lagrangian timescale of particle velocity fluctuations with increasing particle inertia.

3. Acceleration correlations

In isotropic turbulence, the Lagrangian fluid acceleration correlations are defined by the relation

$$A_{\mathrm{L}ij}(\tau) = \langle a'_i(\mathbf{x},t)a'_j(\mathbf{R}(t-\tau),t-\tau)|\mathbf{R}(t)=\mathbf{x}\rangle = a'^2\Psi_{\mathrm{A}}(\tau)\delta_{ij} \tag{26}$$

with $a'^2 \equiv \langle a'_k a'_k\rangle/3$ being the variance of fluid acceleration fluctuations, and $\Psi_{\mathrm{A}}(\tau)$ being the acceleration autocorrelation function.

In view of the kinematic relation $A_{\mathrm{L}ij} = -\mathrm{d}^2B_{\mathrm{L}ij}/\mathrm{d}\tau^2$ and (1), (26) can be rewritten as

$$A_{\mathrm{L}ij}(\tau) = -u'^2\Psi''_{\mathrm{L}}(\tau)\delta_{ij}, \quad a'^2 = \frac{2u'^2}{\tau_{\mathrm{T}}^2} = \frac{a_0\varepsilon^{3/2}}{\nu^{1/2}}, \quad \Psi_A(\tau) = \frac{\Psi''_{\mathrm{L}}(\tau)}{\Psi''_{\mathrm{L}}(0)} = -\frac{\tau_{\mathrm{T}}^2\Psi''_{\mathrm{L}}(\tau)}{2}. \tag{27}$$

The Lagrangian correlations of particle acceleration fluctuations are given by

$$A_{\mathrm{p}ij}(\tau) = \langle a'_{\mathrm{p}i}a'_{\mathrm{p}j}|\mathbf{R}_{\mathrm{p}}(t)=\mathbf{x}\rangle = \langle a'_{\mathrm{p}i}a'_{\mathrm{p}k}\rangle\Psi_{\mathrm{Ap}kj}(\tau).$$

From the kinematic relation $A_{\mathrm{p}ij} = -\mathrm{d}^2B_{\mathrm{p}ij}/\mathrm{d}\tau^2$ and Eq. (7), it is follows that

$$A_{\mathrm{p}ij}(\tau) = \frac{B_{\mathrm{Lp}ij}(\tau) - B_{\mathrm{p}ij}(\tau)}{\tau_{\mathrm{p}}^2} = \frac{u'^2\Psi_{\mathrm{Lp}ij}(\tau) - \langle v'_iv'_k\rangle\Psi_{\mathrm{p}kj}(\tau)}{\tau_{\mathrm{p}}^2}. \tag{28}$$

Expression (28) along with (9) yields the following relations for the particle acceleration variances and autocorrelations:

$$\langle a'_{\mathrm{p}i}a'_{\mathrm{p}j}\rangle = \frac{u'^2}{\tau_{\mathrm{p}}^2}(\delta_{ij} - f_{uij}), \tag{29}$$

$$\Psi_{\mathrm{Ap}ij}(\tau) = (\delta_{ik} - f_{uik})^{-1}[\Psi_{\mathrm{Lp}kj}(\tau) - f_{ukn}\Psi_{\mathrm{p}nj}(\tau)]. \tag{30}$$

With no the mean drift between the particulate and fluid phases, the acceleration variances (29) and autocorrelations (30) become isotropic

$$\langle a'_{\mathrm{p}i}a'_{\mathrm{p}j}\rangle = \frac{a_{\mathrm{p}0}\varepsilon^{3/2}}{\nu^{1/2}}\delta_{ij}, \quad a_{\mathrm{p}0} = \frac{(1-f_u)Re_\lambda}{15^{1/2}St^2}, \tag{31}$$

$$\Psi_{\mathrm{Ap}ij}(\tau) = \Psi_{\mathrm{Ap}}(\tau)\delta_{ij}, \quad \Psi_{\mathrm{Ap}}(\tau) = (1-f_u)^{-1}[\Psi_{\mathrm{Lp}}(\tau) - f_u\Psi_{\mathrm{p}}(\tau)], \tag{32}$$

where $St \equiv \tau_{\mathrm{p}}/\tau_{\mathrm{k}}$ is the Stokes number determined by the Kolmogorov timescale.

In what follows, we consider the acceleration statistics of low-inertia heavy particles, the deviation of trajectories of which from those of the fluid elements may be neglected. In this case, we thus can take $T_{\mathrm{Lp}} = T_{\mathrm{L}}$. Then substituting (23) along with (21) into (31) yields

$$a_{\mathrm{p}0} = a_0\left[1 + \frac{15^{1/2}a_0St(St + \overline{T}_{\mathrm{L}})}{Re_\lambda}\right]^{-1}, \tag{33}$$

where the Lagrangian integral timescale is determined as (Zaichik et al., 2003)

$$\overline{T}_{\mathrm{L}} = \frac{T_{\mathrm{L}}}{\tau_{\mathrm{k}}} = \frac{2(Re_\lambda + C_1)}{15^{1/2}C_{0\infty}}, \quad C_{0\infty} = 7,\ C_1 = 32.$$

Substituting (18) into (31), one can obtain

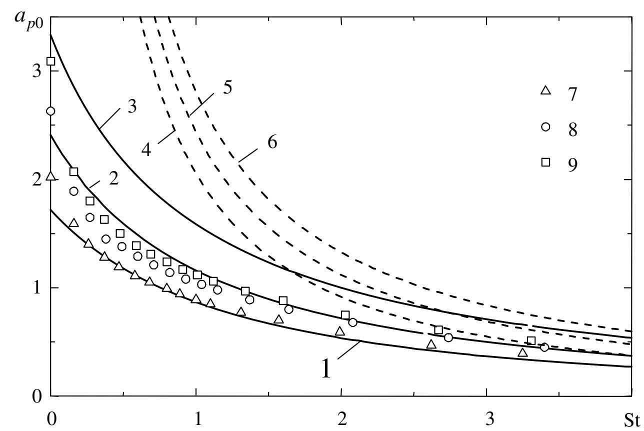

Fig. 2. The normilized particle acceleration magnitude: 1, 2, 3 – (33); 4, 5, 6 – (34); 7, 8, 9 – Bec et al. (2006); 1, 4, 7 – $Re_\lambda = 65$; 2, 5, 8 – $Re_\lambda = 105$; 3, 6, 9 – $Re_\lambda = 185$.

$$a_{p0} = \frac{Re_\lambda}{15^{1/2} a_0 St(St + \overline{T}_L)}. \quad (34)$$

When this relation is compared with (33), it is apparent that (34) obtained when using the single exponential approximation of the Lagrangian fluid velocity autocorrelation function is valid only for high-inertia particles ($St \gg 1$) and it breaks down at small Stokes numbers.

In the limit of high Reynolds numbers ($Re_\lambda \to \infty$), (33) reduces to

$$a_{p0} = a_0 \left(1 + \frac{2a_{0\infty} St}{C_{0\infty}}\right)^{-1}. \quad (35)$$

It is clear from (35) that, in accordance with the classical Kolmogorov similarity hypothesis for small-scale turbulence when neglecting flow intermittency, the effect of Reynolds number on particle acceleration statistics is eliminated.

Fig. 2 shows the normalized particle acceleration magnitude as a function of the Stokes number for three different Reynolds numbers. Predictions based on (33) are compared with DNS by Bec et al. (2006) for the normalized acceleration variance of heavy particles immersed in homogeneous, isotropic, and stationary turbulence with no gravity. Bec et al. (2006) performed a systematic study of particle acceleration statistics in the range of the Stokes number, St, from 0.16 to 3.5 for three values of the Reynolds number, Re_λ. It is clear that, at $St = 0$, the acceleration of heavy particles coincides with that of the fluid. However, the acceleration variance of heavy particles drops very fast with their inertia. As is seen, (33) describes reasonably well the influence of both St and Re_λ on a_{p0}, especially at $Re_\lambda = 65$. Distinctions between the predictions and the simulations at higher Reynolds numbers are explained by neglecting the effect of particle preferential concentration. As was shown in Bec et al. (2006), there are two mechanisms that are responsible for a reduction in acceleration fluctuations with increasing particle inertia: (i) the filtering of fluid velocity differences due to particle inertia and (ii) the local accumulation (preferential concentration) of heavy particles in low-vorticity regions. Because the model presented allows for only the former of these two effects, this leads to an overestimation of the particle acceleration magnitude at small Stokes and high Reynolds numbers when the role of preferential concentration is of particular importance. It is also obvious that, since the acceleration of particles is mainly governed by small-scale turbulent structures, it is sense to quantify the particle inertia in terms of the response time normalized by the Kolmogorov timescale as distinct from the effect of particle inertia on velocity statistics, which is better characterized by the turbulence time macroscale. As a result of such the circumstance, it should be mentioned that inserting the response coefficient (18) into (31) instead of (23) leads at small values of St to an unacceptable behaviour of a_{p0} even in a qualitative sense. It is seen from Fig. 2 that the curves relating to (34) deviate more and more from those regarding (33) as the Stokes number decreases. Thus, to describe the particle acceleration we need using the two-exponential autocorrelation function of the fluid velocity seen by particles, whereas to predict the particle velocity statistics it is sufficient to use the one-exponential autocorrelation approximation of the fluid velocity seen.

In closing make a remark about the role of flow intermittency on the Reynolds number dependence $a_{p0}(Re_\lambda)$. As is clear from (33), this dependence arises mainly from the intermittent correction to the normalized fluid acceleration magnitude $a_0(Re_\lambda)$. Thus, expression (33) supports the conclusion drown in Bec et al. (2006) that the fluid intermittency is responsible of the dependence of a_{p0} on Re_λ.

4. Summary

A simple analytical model for predicting the acceleration statistics of heavy particles dispersed in isotropic homogeneous turbulence is developed. This model takes into account the effect of both Stokes and Reynolds numbers on the particle acceleration. Comparison of model predictions with numerical simulations shows a quite good agreement except for the values of the particle acceleration magnitude at small Stokes and high Reynolds numbers. To improve the model, one has to take into consideration the effect of particle preferential concentration.

References

Bec, J., Biferale, L., Boffetta, G., Celani, A., Cencini, M., Lanotte, A., Musacchio, S., Toschi, F., 2006. Acceleration statistics of heavy particles in turbulence. J. Fluid Mech. 550, 349–358.

Csanady, G.T., 1963. Turbulent diffusion of heavy-particles in the atmosphere. J. Atmos. Sci. 20, 201–208.

Derevich, I.V., 2001. Influence of internal turbulent structure on intensity of velocity and temperature fluctuations of particles. Int. J. Heat Mass Transfer 44, 4505–4521.

Elghobashi, S., Truesdell, G.C., 1992. Direct simulation of particle dispersion in a decaying isotropic turbulence. J. Fluid Mech. 242, 655–700.

Hill, R.J., 2002. Scaling of acceleration in locally isotropic turbulence. J. Fluid Mech. 452, 361–370.

Hinze, J.O., 1975. Turbulence. McGraw-Hill Book, New York.

Mashayek, F., 1999. Stochastic simulations of particle-laden isotropic turbulent flow. Int. J. Multiphase Flow 25, 1575–1599.

Oesterlé, B., Zaichik, L.I., 2006. Time scales for predicting dispersion of arbitrary-density particles in isotropic turbulence. Int. J. Multiphase Flow 32, 838–849.

Pismen, L.M., Nir, A., 1978. On the motion of suspended particles in stationary homogeneous turbulence. J. Fluid Mech. 84, 193–206.

Pozorski, J., Minier, J.-P., 1998. On the Lagrangian turbulent dispersion models based on the Langevin equation. Int. J. Multiphase Flow 24, 913–945.

Reeks, M.W., 1977. On the dispersion of small particles suspended in an isotropic turbulent fluid. J. Fluid Mech. 83, 529–546.

Sawford, B.L., 1991. Reynolds number effects in Lagrangian stochastic models of turbulent dispersion. Phys. Fluids A 3, 1577–1586.

Simonin, O., Février, P., Laviéville, J., 2002. On the spatial distribution of heavy particle velocities in turbulent flow: from continuous field to particulate chaos. J. Turbulence 3 (040).

Squires, K.D., Eaton, J.K., 1991. Measurements of particle dispersion obtained from direct numerical simulations of isotropic turbulence. J. Fluid Mech. 226, 1–35.

Voth, G.A., La Porta, A., Crawford, A.M., Alexander, J., Bodenschatz, E., 2002. Measurements of particle acceleration in fully developed turbulence. J. Fluid Mech. 469, 121–160.

Wang, L.-P., Stock, D.E., 1993. Dispersion of heavy particles by turbulent motion. J. Atmos. Sci. 50, 1897–1913.

Yeung, P.K., Pope, S.B., Lamorgese, A.G., Donzis, D.A., 2006. Acceleration and dissipation statistics of numerically simulated turbulence. Phys. Fluids 18, 065103-1–065103-14.

Zaichik, L.I., Simonin, O., Alipchenkov, V.M., 2003. Two statistical models for predicting collision rates of inertial particles in homogeneous isotropic turbulence. Phys. Fluids 15, 2995–3005.